Proving stuff without freaking out

Micro toolbox

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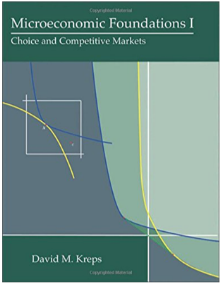
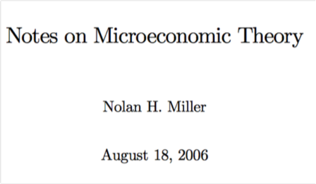
Some tips

1. Sleep and do sports.
2. Carry a notebook with you, write your questions in it.
3. Work in small group.
4. If your friends don't understand your proof, the problem comes from you. Just fix it.
5. Memorize all the definitions, but only the main results.
6. Don't memorize entire proofs, memorize the structure.
7. Everything needs to fit together.

Better resources




Notes on Microeconomic Theory

Nolan H. Miller

August 18, 2006

MWG 3.E.4 p62

Suppose u(.) is a continuous utility function representing a locally non satiated preference relation $\succcurlyeq$ and that $h(p, u)$ consists of a single element for all $p >> 0$. Show that the Hicksian demand function h(p,u) satisfies the compensated law of demand, i.e. for all p' and $p'' >> 0$,
$(p'' - p')[h(p'', u) - h(p', u)] \leq 0$


@MicroeconmicsMemes
marxism
keynesian
austrian
neoclassical

Three steps

1. Read the question again
2. Make sure you understand what each object is (nature and properties)
3. Make sure you understand what you have to show

1. Read again

MWG 3.E.4 p62

Suppose u(.) is a continuous utility function representing a locally non satiated preference relation $\succsim$ and that $h(p, u)$ consists of a single element for all $p >> 0$. Show that the Hicksian demand function h(p,u) satisfies the compensated law of demand, i.e. for all p' and $p'' >> 0$,
$(p'' - p')[h(p'', u) - h(p', u)] \leq 0$

2. What is p? what is h(p,u)? what is h(p',u)?
3. What do you have to show?

Proof architecture: non exhaustive list

Proving an implication

$S \implies N$, for instance S="it is snowing", N="it is cold"
What are we trying to prove?

- Prove directly: assume S, show that we have N
- Prove by contrapositive: $\neg N \implies \neg S$ (if it is not cold it can't be snowing)
  You assume $\neg N$ is true (it is not cold), you show $\neg S$ (it is not snowing)
- Prove by contradiction: similar to contrapositive
  (i) You assume $\neg N$ **AND** S at the same time! (ii) find a contradiction, and (iii) conclude.
  (i) Suppose 77 Fahrenheit (not cold) **and** it's snowing in Davis. (ii) But water is liquid above 32 F at sea level, hence we have a contradiction. (iii) So it cannot be 77F and snowing in Davis.

Proof architecture: non exhaustive list

Proving an implication

$S \implies N$

- Under **no circumstances** should you ever attempt to show $\neg S \implies N$
  (If it's not snowing, it may still be cold)

Truth table

S	N	$S \Rightarrow N$	$S \Leftarrow N$	$S \Leftrightarrow N$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Many other methods, for micro review induction

Proving an equivalence

$p \iff q$, where p="I'm Californian, I'm part of a sports cult and I post my workout sessions on Instagram" and.. q="I do #CrossFit"

You need to prove **BOTH** directions, i.e., $p \implies q$ **AND** $q \implies p$

- Back to proving implications: directly, by contrapositive, by contradiction, etc

Some super important methods

Some classic propositions to prove:

- Set inclusion: $S \subseteq T$
- Equality between sets: $S = T$
- Existence: "if...there exists an $\mathbf{x} \in S$ such that blablabla..."
- Non Existence: "if ...there doesn't exists a $\mathbf{z} \in S$ such that blablabla..."
- Uniqueness: "...and this allocation is unique"

For these, way to prove should click immediately in your head

Set inclusion

S={French morons}, T={morons} , show $S \subseteq T$.
Amounts to showing that **EVERY** element in S is also in T.
Think of $x \in S$ as "p", $x \in T$ as "q": you are back to showing $p \implies q$!

- Directly: let $x \in S$, and let's show $x \in T$
- Contrapositive: let $x \notin T$, and show that x cannot be in S
- By contradiction: let x be a French moron who is at the same time not a moron. That's a contradiction. So every French moron is also a moron.

Note: how would you prove strict inclusion, that is $S \subseteq T$ but $S \neq T$?

Set equality: S = T

Almost always show $S \subseteq T$ **AND** $T \subseteq S$

Existence: "if blabla there exists a function f such that blablabla..."

- Often, by construction: find that function f such that blablabla.
- Can also be by contradiction (less often, e.g. Kreps p. 482)

Non Existence: "if blabla there doesn't exists a g s.t. blabla..."

- Often, by contradiction: assume it exists, then find crazy stuff

Uniqueness: "... a unique function f s.t. blabla"

- Often you assume that there are 2, and you show they are equal
- You assume you have 2 different ones, and you show a contradiction

Some examples

Practice time: how could we start the following proofs?

MWG 2.E.3 p28

If the Walrasian demand function x(p,w) satisfies Walras' law, then for all p and w

$$p.D_w x(p, w) = 1$$

1. Read the question again
2. Make sure you understand what each object is (nature and properties)
3. Make sure you understand what you have to show (what is the formal statement?)

MWG 3.D.2 p51

Suppose that u(.) is a continuous utility function representing a locally nonsatiated preference relation $\succcurlyeq$ defined on the consumption set $X = \mathbb{R}^L_+$. Then the walrasian correspondence is homogeneous of degree 0 in (p,w).

1. Read the question again
2. Make sure you understand what each object is (nature and properties)
3. Make sure you understand what you have to show (what is the formal statement?)

MWG 5.F.1 p150

Let Y be a production set. If y $\in$ Y is profit maximizing for some $p >> 0$, then y is efficient.

Definition efficiency: a production vector y $\in$ Y is efficient if there is no $y' \in Y$ such that $y' \geq y$ and $y' \neq y$.

MWG 3.F.1 p66 Duality Theorem

Let K be a nonempty closed set, and let $\mu_K(.)$ be its support function. Then there exists a unique $\bar{x} \in K$ such that $\bar{p}.\bar{x} = \mu_K(\bar{p})$ if and only if $\mu_K(.)$ is differentiable at $\bar{p}$. Moreover, in this case,

$$\nabla \mu_K(\bar{p}) = \bar{x}$$

... **wtf is a support function?**

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MWG p64: For any nonempty closed set K $\subset \mathbb{R}^L$, the support function of K is defined for any p $\in \mathbb{R}^L$ to be

$$\mu_K(p) = \mathit{Infimum}\{p.x : x \in K\}$$

Bonus: Kreps 10.10 p242 (not used in 200)

A function $u : \mathbb{R}^k_+ \to \mathbb{R}$ is quasi-concave and nondecreasing if and only if its upper-level sets are convex and comprehensive; that is, for all v $\in \mathbb{R}$,

$$\{x \in \mathbb{R}^k_+ : u(x) \geq v\} = CCH(\{x \in \mathbb{R}^k_+ : u(x) \geq v\})$$

Definition Kreps p240: For any set $X \subseteq \mathbb{R}^k$, the **comprehensive convex hull** of X, denoted CCH(X) is the smallest (by set inclusion) set Y that (a) contains X, (b) is convex, and (c) has the property that if $x \in Y$ and $x' \geq x$, then $x' \in Y$